Greg Kuperberg's Lectures on

Very Basics of Quantum Field Theory, String Theory and the AdS/CFT Conjecture

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1 Lecture 1 (12 December 2022)

1.1 Introduction

Simulating a toy-model of a "black hole" on a quantum computer is not the same thing as creating a black hole in the lab. There is a pure CS theory result that comes from AdS/CFT models but that's a fairly ad hoc combinatorics result (the hypothesis are quite ad hoc and not in the aesthetic of pure math).

Consider that you have a Matchbox or Hotwheels collection in 5th grade and say to your friends that you're learning about Nascar. That's similar to the current situation of quantum computing research related to AdS/CFT and actual quantum field theory and AdS/CFT.

1.2 What are quantum field theories?

From the view of quantum field theories, TQFTs (topological quantum field theories) are tools that provide answers as in the context of Turaev's book. But that's not the real thing. What physicists initially had in mind was classical field theories and say Maxwell's equations. What a classical field theory means, is a classical PDE system and "field" as in space-time filling PDE or force field – something that permeates space-time. It's a model for a force field and therefore it is a field theory.

We can agree that randomized computing is a great warmup for quantum computing. You can view it in Bayesian terms. You can think of in terms of probability distributions. Now there is something such as stochastic PDEs. Even Brownian motion – which is a bit hard for the undergrad level – it has a PDE called the heat equation or equivalently it an ODE for a single particle with noise. But you could have a combination of a particle drifting in wind and simulataneously undergoing Brownian noise. If you imagine what happens to a single particle of smoke which follows a wind pattern (Focker-Plank equations). You get a linear PDE from a stochastic non-linear ODE via a "dimension shift". If you'd like to model the probability distribution for Brownian motion in terms of the independent variables is time plus all of the space directions.

Question: In quantum field theories, what are the "fields"? It's the same as classical force fields (say EM fields) and take the quantum version you get the photon/boson field. This requires a quantization process. A quantum ODE is called, in PDE form, the Schrodinger equation. By the way, canonical quantization and deformation quantization are synonymous but not unique. It's not unique even though physicists something pretend it is to because reasons. Often it's true that there is only one reasonable choice of deformation quantization in a particular physical setting. But in full generality of ODE, deformation quantization is thus a better name than canonical quantization.

Classical PDEs often have quantum PDE analogues. Like there is a classical harmonic oscillator, stochastic harmonic oscillator and quantum harmonic oscillator. Or you could take an anharmonic oscillator. That will have a Schrodinger equation. It will be a PDE, for the position of the original particle, but it will be a PDE for the quantum state. But you still think of it as a quantum ODE. You imagine the particle in the anharmonic track as following a quantum superposition of trajectories. In fact, there is a non-quantum classical version – if you started with a noisy linear harmonic oscillator, the corresponding PDE for the probability cloud of the particle is called Ornstein–Uhlenbeck. It looks tremendously like the Schrodinger equation for a harmonic oscillator. The solutions look different, but the equation itself is quite similar.

Question: How are these quantum DEs constructed? Is it just by taking inspiration from the classical case? To some extent yes, but the procedure is better than that – it's called canonical quantization. It's called canonical but it's not necessarily unique. The classical Hamiltonian is replaced by a quantum Hamiltonian that you now put in the Schrodinger equation. The same thing would happen for a noisy version. But in this case there wouldn't be a single equation because there can be different noise models. You could have noise terms for different reasons, like for a solar system, and then you'd have more than one possible Schrodinger equation for it.

But in both cases, noisy or quantum, there would be a way to recover the classical limit. In the quantum case, as you send $\overline{h} \to 0$, it will look more and more like the classical system, similar to what would happen if you shrink the noise term for a noisy PDE in the classical case. Anyway, the harmonic oscillator is quite teachable for the dimension shift of the math. The classical harmonic oscillator is an ODE where the only independent variable is time – in that sense it's one-dimensional – just time dimensional. The quantum counterpart to the harmonic oscillator is the Schrodinger equation for harmonic oscillator.

So we have seen that quantum field is basically the study of a special class of ODEs and PDEs. But also quantum probability comes into the picture. It comes in the same way as noise would for a PDE. What we want to emphasize is the "dimension shift" that occurs for quantum mechanics and it's similar to the dimension shift for stochastic ODEs from ODEs. This comes from the fact that we're just modelling position but probability distribution for a position. To review, this is identical dimension shift for noisy ODEs.

Suppose you had a single particle in space that's an ODE with 3 dependent components, if it's classical mechanics. Then the Schrodinger equation has 3 + 1 independent variables. If now you say have a helium atom, it has 3 particles treating the nucleus as one particle, and you took a classical solar system model – you have a star and two planets. Taking it as a classical system, what is the number of independent and dependent variables? So you need position and momenta for the two planets, fixing the star in the middle. So 6 + 6 dependent variables. The number of independent variables is just 1, time. It's still an ODE with more and more components.

Now let's take a quantum version of the same thing. The Schrodinger equation for helium atom. How many dependent variables are there now? You have a joint quantum state. Let's just take the standard Schrodinger's equation for Helium atom. One thing that's a little subtle is that position and momentum are dual measurements of the same state – so you get a PDE only in terms of PDE and derivatives. But you still have position – so that's 6 variables – these are now independent. Another 1 independent variable is time. So 7 independent variables. The dependent variable is amplitude-valued. For any Schrodinger equation, there's a single complex valued dependent variable – the amplitude density.

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)\right]\Psi(x,t)$$

In the above SE for a single particle, the independent variables – there are 4 of them, one for t and 3 for x. The velocities get converted to partial derivatives. In the classical version, x would be dependent and t would be independent. But also the velocity v or momentum p would be a separate independent variable. That is lost when you make the quantum version – because position and momentum are actually measurements of the same quantum state – they're just non-commuting measurements. The number of variables that get converted from dependent to independent also get cut in half in a subtle way. For a two particle SE, it looks something like $\psi(r_1, r_2, t)$. So the gist is that in stochastic or quantum form, more and more independent variables are accumulated.

So for we have been doing quantum mechanics is and that's just quantum ODEs. Everything is still rigorous. But also quantum ODEs are already PDEs – linear PDEs with a growing number of dimensions. So indeed quantum mechanics is a rigorous topic and so is the topic of stochastic ODEs.

When you have a QFT, you should start with something that's already a PDE and give it the same dimension explosion to make a quantum or stochastic version. So PDEs – if you try to interpret them as ODEs – they are infinite-dimensional ODEs. Question: Infinite-dimensional in what sense? You have a DE on a function space. Take the Maxwell's equation – the electric field at any point in time has infinitely many degrees of freedom – therefore you can interpret the Maxwell PDE system as ODE in an infinite-dimensional function space. But what quality of functions should we use? Is it like $C[0,1]$? That's a real question and it's subtle. Although if you had some theorem for a given PDE that what started smooth stayed smooth as time evolves then you could say that the function space involved was smooth functions. It's an infinite dimensional vector space inevitably. If you want to read a PDE as an ODE, it's an infinite dimensional ODE.

So what no one knows how to do rigorously is to make either a stochastic or quantum version of something that in deterministic form is already a PDE. You need an analogous explosion of degrees of freedom. (Note: Given a linear ODE, the set of solutions form a vector space with finite dimension. However, a linear PDE (like the heat equations) has a set of solution that form a vector space with infinitely many dimensions.)

Say, in the Maxwell's equations you would want something like a quantum state that assigns a quantum amplitude or density to every possible EM field. To have a model for arbitrary superpositions of entire EM fields. And there is not really a rigorous version of that. The dimension explosion is so bad that there is not really any good math defintions for it. OTOH, there certainly are trustworthy standards. There are some elusive standards for a QFT to be healthy and where you can expect trustworthy solutions. So quantum field theory is basically the study of PDEs with specific restrictions taken in quantized form.

1.3 What's Witten work and what's Chern-Simon's theory?

From Wikipedia,

The action S of Chern–Simons theory is proportional to the integral of the Chern–Simons 3-form

$$S = \frac{k}{4\pi}\int_M \operatorname{tr}\left(A \wedge dA + \tfrac{2}{3} A \wedge A \wedge A\right)$$

This terminology doesn't look like a differential equation but it is. An ODE or a PDE can be put in Lagrangian form and that is not a quantum thing to do. That's a classical concept. Where, what you're saying, if you put an ODE or PDE in Lagrangian form, you're viewing it as a minimization problem and the PDE equation itself comes out itself of the critical point equation. The Chern-Simons 3-form is in Lagrangian form. If you integrate it over the manifold M, that is a specific numerical functional over the field A. What's meant by field? It's some tensor field on the manifold M. What's this manifold in the first place? It's the PDE manifold, on which you set up the manifold. Like atmospheric scientists set up PDEs for earth's air considering the surface of the earth as a manifold. Now there's a specific quantum field theory that is a quantum version of a classical field theory, which is called Chern-Simons. The Lagrangian formulation of the equation of a certain tensor field A, where the domain of A (not the domain of the PDE, which is all possible values of the field A) is some 3-manifold.

In classical Chern-Simons, you would just take this functional S, and it's the Lagrangian formulation of some PDE system and you'd write the corresponding PDE for the condition of being at a critical point of S, where the partial derivative of S w.r.t A vanishes. Then quantum Chern-Simons is the quantum version of the same thing. Now in Witten's famous paper from the 80s, he took this Chern-Simons PDE – he made the quantum version and then he demonstrated at a standard that is satisfactory for physics that this quantum field theory is topological. (Coobordism comes in later, which is say a 3-dimensional connection between 2-dimensional surfaces.)

When Witten says that this Chern-Simons is a topological quantum field theory, what he means by TQFT is not a definition of a type of mathematical object. What he means is that the QFT has topological properties. He means that the answers that you get out of this QFT only depend on the topology of M and maybe things like knots and links drawn in M. While in math there's a rigorous thing called a topological quantum field theory (whereas QFT has no rigorous definition in math). What quantum field theorists would establish as properties of a quantum field theory – it's taking that and changing it to a definition of an object. Question: Are these objects realistic or is it just a simplification? No, no. Witten showed that the solution to quantum Chern-Simons – all of the information is purely topological. So it's a full summary of the properties of Chern-Simons quantum field theory when you solve it.

Question: What does this Chern-Simons quantum field theory physically describe? It's an unrealistic quantum field theory. If by realistic you mean how do quantum field theorists conceive of it, then we can discuss that. If by realistic you mean when does it show up in real life, then it was not originally promised as realistically describing anything in real life. First of all the dimension is wrong and geometry is wrong. It does not a priori represent any physical system at all. There's something wrong with it if you want to literally take it as physics. First of all, M is not 4-dimensional spacetime, it's a 3-manifold. So the total dimension of this QFT is 3; what does this correspond to in real life? Who lives in a spacetime that is a 3-manifold with some topology? Nobody. Second, spacetime has a metric which is used for realistic quantum field theories. In space, there is distances and in spacetime you have the relativistic version of the same thing – you have the relavistic metric. But in this quantum field theory there is no metric. So for this to be realistic, someone should live in a 3-dimensional spacetime with no metric. Nobody does.

Question: So if this isn't real physics, then why was Witten concerned about it? Because he is at heart a mathematical physicist. He is a physicist but a mathematical one, who is interested in gedanken experiments as needed in unrealistic settings. We can make an analogy on this point. Take the fluid equations – the popular choice is Navier-Stokes. (The Navier-Stokes equations were real popular in the UC Davis math department when Greg came here.) The Navier-Stokes equations are realistic physics but almost so because even though are smooth in the large scale, in the small scale you have molecules and it's discrete. But in the large scale, the NS equations have a lot of credibility. So let's call that realistic physics. It was quite common to do simulations of the Navier-Stokes equations on a 3-torus. There's also time, but space is a 3-torus – because you have a box and you don't want to have to worry about the boundary, and so you just make periodic boundary conditions. Is partly realistic or entirely unrealistic? There's things to learn about real life from a thought experiment like that. Insofar as Chern-Simons ia QFT with interesting behaviour we can call it physics, but there is an admission that it has unrealistic features. What was spacetime is now just 3-dimensions rather than 4 and it also just doesn't have a metric at all.

Question: You said that in quantum field theory you want to assign amplitudes to different fields – I suppose you have fields in different points in space and you're assigning amplitudes to those. A: No, you have fields for all of space and for every example there is a quantum amplitude. In quantum Chern-Simons you can do this and that assignment works too well, when quantum field theories are put in Lagrangian form. Because all you're supposed to do is take the imaginary exponential of the Lagrangian action. So for a given A (which is a tensor field on all of M), its ampltude is e^{iS}. The catastrophe is that if you want to do any interesting quantum probability, you should take integrals. But integrating over the space of all choices of A is what people don't know how to do, in any manner that's both sane and relevant, because the domain of the space of integration is just too big.

So a physicist would look at this and say that this is a topological quantum field theory, because the answers are topological invariants and satisfy as properties what you would have given as axioms. Question: Do physicists even care about these toy models? A: Well, Witten's paper was opportune because actually TQFTs as a correct effective field theory for anyons. "Effective" means an approximation. The vast Schrodinger equation for some appropriate 2D system will more or less simulate the same sort of topological quantum field theory that you get from Chern-Simons.

Question: How do you convince yourself that e^{iS} is the appropriate amplitude for Chern-Simons? A: Once you have a Lagrangian formulation, the die is cast for canonical quantization. All the classical PDE is, is the critical points. The same PDE can have two different Lagrangians for it, just as in ordinary calculus can have critical points at the same places. When you have a PDE system, there will be something called (a) not (b) Lagrangian formulation. It's not unique. Then as a method of quantization, you can have a declaration, that whatever S is for Lagrangian version of PDE system, it's imaginary exponential could be the chosen quantum density for quantization. That's a definition, actually.

Question: So what do mathematicians or physicists really mean when they talk about trying to make quantum field theory rigorous? A: The idea is to come up with some rigorous math that matches the black magic math that quantum field theorists do, that on the other hand is clearly important. Q: Is it just finding a rigorous version of PDEs on infinite dimensional function spaces of some kind. A: That's too general actually. No, the task of quantum field theory is adding quantum probability to what at the beginning is a classical PDE system. And actually stochastic PDEs have the same crisis – even so much as adding noise to classical PDEs leads to the same crisis of rigor as QFT does, even though it's not quantum. Q: So will solving the stochastic problem help in solving the quantum problem? A: It will help in defining and one will indeed help the other, and to some extent they're the same question. The math crisis is similar and the reason for the crisis is similar.

1.4 Yang-Mills QFTs

There is an important class of quantum field theories, called Yang-Mills, which are in fact part of the Standard model and in $3 + 1$ dimensions – quantum Yang-Mills is well, quantum, and Yang-Mills has a quantum version. This is used to explain quarks and gluons. All the Yang-Mills is, is a matrix version of Maxwell's equations. Because Yang-Mills is matrix version of Maxwell's equations, (1) It is non-linear and harder to solve, and (2) some of the solutions have quite different behaviour. Classical Yang-Mills looks a whole lot like Maxwell's equations. So anyway, there is a quantum Yang-Mills – there exists a quantum expansion of Yang-Mills in 3+1 dimension. Now Yang-Mills also has a 4+0 dimensional version for Riemannian 4-manifolds and is the basis of Donaldson theory, and the physicists will say field theory for that – but it's actually stochastic rather than quantum. Confirmed with Tudor Dimofte, that for 4+0-dimensional Yang Mills, there is a similar S, which is some other integral over a manifold M, which in the case Yang-Mills does depend on a metric. And then in 4+0 dimensions, you don't take the imaginary exponential of S but rather than real negative exponential of the Yang-Mills S instead.

Then you get a stochastic model, where each example field has a probability density rather than an amplitude. Nobody knows how to integrate that rigorously, although they can do it reliably. Question: Is Donaldson theory for 4+0-dimensions rigorous? A: It is, because it just a remnant of the stochastic Yang-Mills on a 4-manifold. Q: So the problem arises in 3+1 dimension? A: No, you the same crisis of rigor whether you take stochastic Yang-Mills in 4+0 dimensions or quantum

Yang-Mills in 3+1 dimensions. Physicists have a dozen methods and there's a black magic feature of all of them.

Here's where the story begins. Here's the sort of thing that people could say even in the early days of quantum field theory. How do you get photons from Maxwell's equations? Maxwell's equations splits in harmonic oscillator. It's an ocean of harmonic oscillators. Maxwell knew that and that's why he found wave solutions. So quantumly, let's just take the same splitting for harmonic oscillators of Maxwell's equations and just take them as separate quantum harmonic oscillators. Is that a solution or a definition, you might ask. Well, both. The physicists would take that as a method of treating the quantum Maxwell's equations whereas the mathematicians would say – what are you talking about? We can take this as a definition – we can take each separate harmonic oscillator living in Maxwell's equations and turn that into a quantum harmonic oscillator. But as a method of solution, you didn't really set up anything rigorously to solve. That is, Maxwell's equations split up into harmonic oscillators. That's a theorem, classically. So we take that theorem as an approach to quantum Maxwell's equations and say that they split up into quantum harmonic oscillators.

Now each harmonic oscillator has an energy level which is an integer. Well, plus a half as sometimes people say. But for now don't worry about that extra half. There's an integer level for the quantum harmonic oscillator. Take that integer for the QHO and interpret it as a count of photons. That's what they call a photon occupation number. In other words, how many photos exist that are undergoing that specific oscillation. And that's the definition of a photon. If you think of a system of non-negative integer counts then you can interpret that as counting objects. So that's what you do. You have an ocean of harmonic oscillators in Maxwell's equations. You interpret them as separately quantum. Each one has a non-negative energy level which is a non-negative integer and you interpret that as the number of photons. The energy levels correspond to photons. This argument is called **second quantization**. You're making particles out of something where nothing had been quantized first – it was just waves on waves. You had a quantum state on something that was already waves. And out of it you get particles, from the harmonic oscillator route – that's called the second quantization argument. So this argument really has the ring of truth to it. It also agrees with real life, but as maths it's a little nuts.

Q: So are all elementary particles solutions to some differential equations over fields as in these energy levels? A: Yes, but partly by derivation, and partly just by some standalone interpretation that sounds more like a definition. But in fact, in other cases, you can make other reliable predictions of new particles (non-fundamental) and there will more or less rigorous meaning. In some cases, it will be very similar. For instance, there is a particle called a phonon. You "make" them, on paper, that's very similar to the 2nd quantization justification of photons. Instead of Maxwell's equations, you start with the sound equation in a crystal at low temperature. Now the sound equation is non-linear, actually, but quiet amounts of sounds it's very close to linear. So just take linearized sound equation. So there is a sound equation for material that depends on distortion and stress. It decomposes into HOs – you can quantize each one of them – and you will get a non-negative levels and you will get a count of phonons. So you get a prediction that in a crystal – there will be particles that bounce around in a crystal in a kind of 3D billiards kind of way – and those particles will be phonons. And that prediction is correct – the prediction will also tell you the energy and velocities of each phonon types.

Q: What is the distinguishing factor between quasi-particles and elementary particles? Both seem to be solutions to some differential equations. A: In high energy physics, there is no distinction. In condensed matter physics, we're basically always with a giant Schrodinger equation for an Avogadro number of particles. And you're given fundamental particles from on high, from a restricted list.

Most of the times those particles are electrons, photons and nuclei – and the nuclei are complicated lists – actually they have been protons and neutrons and we'll call them fundamental too. In high energy theory, OTOH, this distinction has no place – because everything comes from the underlying quantum fields. In high energy physics, everything is quasi-particles but one particle may be more fundamental than another – say quarks are more fundamental than photons. The quasi-particle construction in high energy physics is then simply the particle construction. But it's also used in reversible form. Given a particle type, you can describe it in field terms. This is occasionally useful in condensed matter too – which is what we can called quasi-fields. If you have a dilute gas, like they do in the laser trap experiments, you can truthfully imagine a quasi-field for Helium, which would be exactly the argument for existence of photons and phonons but exactly in reverse. However, this is not a very popular approach in low-energy matter physics.

The backward reasoning is a particularly important case. If you start with what looks like a classical field theory and then quantize it, then you will only get bosons – so the construction for fermions is really done in reverse. There is an electron field that is treated as the EM field for photons. But instead of having a commuting limit a fermion field has an anti-commuting limit – it's just generic quantum in a different way. It is the quasi-particle construction in reverse. That's what people mean by fermion fields in high energy physics. Q: Is this all that high energy theorists think about? A: The answer is yes, that's all they did think about, until they had justification to think about string theory instead. What happens in string theory, what you could either view as a method of solution or a dual description for a quantum field theory in terms of Feynman diagrams. At first, they showed up as a method of solution. Eventually they have the character of a dual description. Feynman diagrams are graphs and the proposal is, in this dual description, to replace graphs by 2-dimensional surfaces. In fact by conformal surfaces, by complex curves, by Riemann surfaces. In this dual description, that's string theory. The particles, at any single point in time, get replaced with loops. They don't really care that much about loops – what they really care about is what they replace Feynmann diagrams with. The answer is with Riemann surfaces. Q: How successful is that replacement? Is it rigorous? A: No. Like all of QFT, there are consistency checks and glimmers of rigor. However, the topic as a whole is not rigorous.

1.5 What's string theory?

There's also this other crisis – which is that, in QFT there's these dual descriptions – on one side, a quantum PDE given in Lagrangian form like S – and OTOH the dual description in terms of Feynman diagrams, also taken in superposition. String theory doesn't have both sides, not in any known way – on one side the Feynman diagrams are replaced by Riemann surfaces which are called worldsheets of strings. PDEs have no analogues in string theory. However, they have argued successfully although not rigorously, that there is a classical limit – general relativity. So string theory is a non-rigorous but credible model for quantum general relativity. Replacing Feynman diagrams with Riemann surfaces is called string theory. So string theory is a non-rigorous but credible model for quantum general relativity.

What is a Feynman diagram? It's a spacetime history of some particle collisions. And a word that people would use for spacetime history of collisions – is a world-path – like your path in life. So they use the phrase "world-sheet", for the equivalent thing in string theory, where the Feynman diagram graph is replaced by a Riemann surface. But that's making an analogue of only one part of string theory – only the Feynman diagram side. OTOS, you have a quantum PDE formulation, like in the Chern-Simons case. So the counterpart in string theory, where you would make a Lagrangian formulation where the strings – worldsheets or Riemann surfaces are the solutions rather than definitions – people do not know how to do that in string theory – not in the correct way. But that's a bigger ask than simply describing a classical limit, and indeed string theory has GR in the classical limit – reliably but not rigorously. The reason that string theory remains possible is that it's the only credible candidate for quantum gravity, in dimensions 4 and up – that actually resembles sane mathematics. Anything else that people can think of, doesn't resemble particularly rigorous mathematics.

Question: By the way, is "string theory" singular or plural? Are there different kinds of string theories? A: So, in the fundamental definition there are six string theories – one that's bosonic and not particularly healthy and five that are supersymmetric. They are different. But there are credible arguments that they grow together and that they're dual descriptions of one common thing. There's a separate problem that has many candidates – more than 5 – those 5 candidates have grown together into the same thing and the 5 have become 1. But in another respect, there are many candidates. Anyway, string theory is the only viable candidate in 4 and up dimensions. In other proposals, the discussion is just unstable and it doesn't seem like anything consistent is coming. But even then the theory doesn't exactly like 4 dimensions, it like 10 = 9 + 1 dimensions. Where did the other 6 go? The proposal is that they're very small. Vibrations that momenta, and activity, in these small dimensions, will be manifested as these short distance fields. You don't even just get quantum gravity, you get the whole kabuttal – you'll get EM and quarks and all that jazz as the remnant of the six dimensions that are too small to see. You have quantum gravity and the only game in town for several decades is 9 + 1 dimensions. Then you must have a story for the other 6 dimensions. Well, even if they are too small to see, they will have physical effects, and what will the physical effects be – let's hope that it's something that we see in real-life also. Then there's some reasonable hope that it will be the other forces besides gravity, so that you'll explain everything at the same time. Unfortunately, the geometry of these 6 tiny dimensions is negotiable and gives you a zillion different effective QFTs for what you see in 4 dimensional spacetime. They say rolled up dimension but what they mean is Planck scale. By the way, spacetime is considered to be a manifold and we're talking about its dimensions.

In lot of discussion of string theory, people will often mockingly say, "6 rolled up dimensions of Planck length? Yeah, right. Why are you imposing this on us to make us believe this?" The answer is that they're pursuing the only way that looks like sanity in the effort to combine QM and GR. Then the math throws them into 10 dimensions and they need to do something with the other 6. Then it's the only road to follow. Question: Is it 10 or 11? I heard 11 in some places. A: So, Feynman diagrams arise in what's called perturbative QFT – they arise in power series expansions for the answers. Unfortunately, these power series expansions will often have zero radius of convergence. If you have a smooth function, even if it's not analytic, you still get accurate approximations from the power series for a while – that's actually how QFT is used. String theory only has its perturbative form and in the thing that string theory, as defined, is a power series of, people keep guessing that and for some reasons the dimension changes 10 to 11. Their predictions of 11 or 10+1 dimension geometric behavior is from things that people can compute or conjecture.

1.6 AdS/CFT Conjecture

In the Wikipedia page, on one side string theory is mentioned and on the other side, quantum field theory is mentioned. The proposal is that you'll have some infinite spacetime that you can call the region at infinity – the celestial limit. There is a proposed equivalence between string theory in the middle and a certain type of QFT at infinity. The fact that the thing at infinity is only a QFT and not as punishing as a string theory – is considered quite promising for studying what string theory is or what it should be. Question: Usually they say, it's a correspondence between bulk and boundary where the boundary is 1 dimension lower. A: Yes, but because the boundary is the night sky rather than a wall. Q: So does it inform you about string theory? A: The idea is that the limit of – if you call what's in the middle the universe and then the limit at infinity the celestial sky – the idea is that what's in the celestial sky can be understood (maybe) better in terms of what's in the middle. And this proposal will in principle let you study either to define or study what is in the middle. Q: Can we get information about boundary by studying the bulk and vice versa? A: Yes. Q: Are these specifically considered over AdS space? A: Yes often, but not necessarily. Our universe is de Sitter (with galaxy scale granola) rather than AdS. As you beyond the galaxy scale, it appears to converge to nice geometric de Sitter. Q: Is AdS/CFT even useful for us? A: If you're struggling to define M-theory or interpret string theory, it's useful for that. That's where it really came from. It's one of these gedanken experiment things, as the geometry is wrong. But it's like analyzing Navier-Stokes on 3-torus and you might just end up learning something that's relevant to real life.

Now there's another replacement you can do that's even more radical. You can replace the bulk by a finite tensor network (even a small one) and the celestial boundary by the boundary of the tensor network. And then you can have a joint paper with a QI theorist. But then you'd be blowing away all of the warning signs that we've been talking about for two hours about what quantum field theory is meant to be. Most QFTs are not topological BTW, and they'll depend on a metric and they'll be geometric. You'll not the same sort of axioms as you might have discussed as a defintion of TQFT.

Q: Are information theorists in these smaller kinds of toy models? A: If you're an IT like Daniel Harlow, then you might be better versed in actual AdS/CFT. In fact, Harlow's work is fairly loose compared to the quite punishing arena of quantum PDEs, much less string theory which is even harder. But still his papers have a less of toy feel, because he is actually from the theoretical physics community for real. When he splits up spacetime, the bulk into two regions, he is really describing a universe – he has in mind a universe and not a tensor network.

Q: So are the CS theory results on this based on the tensor network concept? A: Yes, else how are you going to involve QC theorists rather than tensor networks? Circuits too if you like. Here there will be holography, which says that the quantum information in the celestial sphere is a holographic encoding of what's in the bulk. So you replace holography with quantum codes. Q: Is holography related to quantum error correcting codes? A: Yes, if you wanted a toy model – they may or may not be error correcting – but they'd be something like quantum codes. To pull in the QC theorists, you rasterize everything. In so doing, you deliberately ruin the functional analysis feel of QFT done the honest way.

Now, to be fair, in the TQFTs, since the answers are topological – the rasterization is not a ruination. It's mathematically identical. The TQFT axioms of Segal and others and the solutions by Reshetikhin-Turaev may be missing the point but it isn't missing the answer. But if you are going to rasterize AdS/CFT, everything should agree that it's a toy version.